

# An Impact of the Metrics Probabilistic Distributions on the Spatial Geometry of the Universe in Quantum Model

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## Abstract

It is shown that the homogeneous and isotropic Universe is spatially flat in the limit which takes into account the moments of infinitely large orders of probabilistic distribution of a scale factor with respect to its mean value in the state with large quantum numbers. The quantum mechanism of fine tuning of the total energy density in the Universe to the critical value at the early stage of its evolution is proposed and the reason of possible small difference between these densities during the subsequent expansion is indicated. A comparison of the predictions of the quantum model with the real Universe is given.

**Key words:** constraint system quantization, quantum cosmology, spatial geometry.

## 1 Introduction

Among the puzzles of the classical cosmology based on the equations of general relativity the flatness problem is one of the most important [1, 2, 3]. The model of inflation [2, 4] ensures a strict equality  $\Omega = 1$  within the framework of classical cosmology due to the hypothesis of the De Sitter (exponential) expansion of the early Universe. The standard  $\Lambda$ CDM model which includes the inflationary scenario solves one fine tuning (flatness) problem, but leads to a number of the new ones (the coincidence between the contributions from dark matter and dark energy to the total energy density, the smallness of the vacuum energy term and a requirement for fine tuning of it) [5].

Measurements of the anisotropy of the cosmic microwave background (CMB) make it possible to determine the total energy density  $\Omega$  and its components in our Universe. The available astrophysical data indicate clearly that the modern Universe is very close to be spatially flat [6, 7, 8, 9, 10, 11]. The results of the WMAP experiment together with the evidence from the 2dFGRS research and observations

of type Ia supernovae reveal a systematic small deviation of the total energy density of the Universe in the direction where it exceeds a little the critical value [10]. The most accurate data on the spectra of the CMB fluctuations [12] were obtained in the WMAP experiment [11, 13, 14]. The position of the first acoustic peak measured in this experiment, which provides the evidence for spatial geometry, gives the total energy density equal to  $\Omega = 1.003^{+0.013}_{-0.017}$  [11].

Since a fitting of the values of the free parameters is performed in multiparametric space, then the possible range of these values is preassigned in the context of certain assumptions. In this meaning the interpretation of the WMAP data on the existence and contributions from separate components in the total energy density  $\Omega$  is model-dependent and may be inadequate to real physical processes in the expanding Universe. Moreover, secondary effects, such as the Sunyaev-Zeldovich effect [15] on the observed CMB anisotropy for galaxy clusters at redshift  $z \lesssim 1$  [5, 16] and foreground contamination of the CMB power spectrum from an early epoch of reionisation at  $10 < z < 20$  [17], might be underestimated. The search for theoretical models which would provide the higher level of flexibility with respect to observational cosmology in comparison, e.g., with the standard  $\Lambda$ CDM approach is required.

In the present paper a question about the spatial geometry of the Universe is analyzed on the basis of quantum cosmological model proposed in [18, 19, 20, 21]. It has been demonstrated that the homogeneous and isotropic Universe is spatially flat in the limit which takes into account the moments of infinitely large orders of probabilistic distribution of a scale factor with respect to its mean value in the state with large quantum numbers. The quantum mechanism of fine tuning of the total energy density in the Universe to the critical value at the early stage of its evolution is discovered and the reason of possible small difference between these densities during the subsequent expansion is indicated.

## 2 Quantum Model

As is well known (see, e.g., [22]), quantum theory adequately describes properties of various physical systems. Its universal validity demands that the Universe as a whole must obey quantum laws as well. Since quantum effects are not *a priori* restricted to certain scales [23], then one should not conclude in advance, without research into the properties of the Universe within the theory more general than classical cosmology, that its space-time structure at large scales will be classical automatically (the motivation to develop quantum cosmology see in [24, 25, 26]).

The results of the investigations presented in this article are based on quantum cosmology at the heart of which lies the method of constraint system quantization proposed by Dirac [27] with the addition of the idea of introduction of an additional medium or source which determines the reference frame in the Einstein-Hilbert Lagrangian [18, 21, 28, 29, 30].

As it has been demonstrated in [18, 19, 20, 21] in quantum theory the homogeneous, isotropic and spatially flat Universe filled with the primordial matter in the form of a uniform scalar field  $\phi$  is described by the time-dependent Schrödinger type equation

$$i \partial_T \Psi = \hat{\mathcal{H}} \Psi, \quad (1)$$

where

$$\hat{\mathcal{H}} = \frac{1}{2} \left( \partial_a^2 - \frac{2}{a^2} \partial_\phi^2 - a^2 + a^4 V(\phi) \right) \quad (2)$$

is a Hamiltonian-like operator,  $V(\phi)$  is a potential energy density of the field  $\phi$ . Here and below we give all relations between dimensionless quantities. The length is taken in units of the modified Planck length  $l_P = \sqrt{2G\hbar/(3\pi c^3)} = 0.744 \times 10^{-33}$  cm, the density is measured in units of  $\rho_P = 3c^4/(8\pi G l_P^2) = 1.627 \times 10^{117}$  GeV cm<sup>-3</sup>, and so on.

The wavefunction  $\Psi$  depends on a cosmological scale factor  $a$ , a scalar field  $\phi$  and time coordinate  $T$  related to the synchronous proper time  $t$  by the differential equation  $dt = a dT$ . When deriving Eq. (1) from the principle of least action, “time”  $T$  is introduced in the theory by means of the coordinate condition and takes the role of the additional variable which describes the medium that defines the reference frame [18, 21]. In the semi-classical approach this variable describes the source of the gravitational field in the form of relativistic matter of an arbitrary nature. Equation (1) has a particular solution with separable variables

$$\Psi = e^{\frac{i}{2}ET} \psi_E, \quad (3)$$

where the function  $\psi_E$  is defined in the  $(a, \phi)$  minisuperspace and satisfies the time-independent equation

$$\left( -\partial_a^2 + \frac{2}{a^2} \partial_\phi^2 + U - E \right) \psi_E = 0, \quad (4)$$

while

$$U = a^2 - a^4 V(\phi) \quad (5)$$

can be interpreted as an effective potential. We note that in the limiting case  $E \rightarrow 0$  Eq. (4) formally turns into the Wheeler-DeWitt equation for the minisuperspace model [31].

Since the Hamiltonian-like operator (2) contains an isotropic oscillator operator with respect to the variable  $a$  as a subsystem, it is convenient to choose the integration with respect to this variable with a unit weight function. Using Eq. (1) and taking into account that the operator (2) is Hermitian, we obtain the equation which describes the evolution of the mean value of some physical quantity represented by the operator  $\hat{A}$  in “time”  $T$ ,

$$\frac{d}{dT} \langle \hat{A} \rangle = \frac{1}{i} \langle [\hat{A}, \hat{\mathcal{H}}] \rangle + \langle \partial_T \hat{A} \rangle, \quad (6)$$

where  $[\hat{A}, \hat{\mathcal{H}}] = \hat{A}\hat{\mathcal{H}} - \hat{\mathcal{H}}\hat{A}$ , and the brackets denote the averaging over the state  $\Psi$  normalized in one way or another (see below). Introducing, as usual[32], the operator  $d\hat{A}/dT$ , such that

$$\langle \frac{d\hat{A}}{dT} \rangle = \frac{d}{dT} \langle \hat{A} \rangle, \quad (7)$$

we arrive at the Hiesenberg-type operator equation

$$\frac{d\hat{A}}{dT} = \frac{1}{i} [\hat{A}, \hat{\mathcal{H}}] + \partial_T \hat{A}. \quad (8)$$

Setting  $\hat{A} = a$ , from Eq. (8) we find

$$a \frac{da}{dt} = -\hat{\pi}_a, \quad (9)$$

where  $\hat{\pi}_a = -i \partial_a$  is the momentum operator canonically conjugate with  $a$ . The operator equation (9) is equivalent to the definition of the momentum  $\pi_a = -a da/dt$ , canonically conjugate with the variable  $a$  in classical cosmology [2, 18, 21].

Setting  $\hat{A} = \hat{\pi}_a$ , we obtain the equation of the evolution of the momentum operator  $\hat{\pi}_a$  in time  $t$

$$a \frac{d\hat{\pi}_a}{dt} = \frac{2}{a^3} \hat{\pi}_\phi^2 + a - 2a^3 V(\phi), \quad (10)$$

where  $\hat{\pi}_\phi = -i \partial_\phi$  is the momentum operator canonically conjugate with  $\phi$ . This equation is the quantum analog of the canonical equation which determines the time evolution of the momentum  $\pi_a$  in classical cosmology. The momentum of the scalar field, as is well known, equals  $\pi_\phi = \frac{1}{2} a^3 d\phi/dt$ . The quantum analog of this relation follows from (8) at  $\hat{A} = \phi$  as well. It has a form

$$a \frac{d\phi}{dt} = \frac{2}{a^2} \hat{\pi}_\phi. \quad (11)$$

Using the relations (9) – (11), one can obtain the quantum analogs of all equations of general relativity for the homogeneous and isotropic Universe filled with the uniform scalar field and the relativistic matter.

### 3 Choice of Physical States of the Universe

According to (4) the quantum state  $\psi_E$  depends on the form and numerical value of the potential energy density of the scalar field  $V(\phi)$ . In the range of values of the field  $\phi$ , where the density  $V(\phi)$  is the positive-definite function, the effective potential  $U$  (5) as a function of  $a$  at the fixed value of  $\phi$  has the form of a barrier. In this case, the Universe described by Eq. (4) can be both in continuum states with  $E > 0$  and quasistationary ones which correspond to complex values  $E = E_n + i\Gamma_n$ , where  $E_n > 0$ ,  $\Gamma_n > 0$  and  $\Gamma_n \ll E_n$ ,  $n = 0, 1, 2, \dots$  is the number of a state [18, 19, 20, 21].

Quasistationary states are most interesting from the physical viewpoint, since the Universe in such states can be described by a set of standard cosmological parameters accepted in classical cosmology (details see in [21]). At the same time the predictions of the quantum model can be compared both with the predictions of the standard classical cosmology and the data from astronomical observations.

It can be demonstrated [21, 33] that the wavefunction of a quasistationary state considered as a function of  $a$  at the fixed  $\phi$  has a sharp peak and is concentrated mainly in the region limited by the barrier  $U$ . Then, following Fock [34], one can introduce an approximate function  $\tilde{\psi}_E$  which is equal to the exact wavefunction  $\psi_E$  inside the barrier and vanishes outside it. Since the phase of the exact wavefunction  $\psi_E$  outside the barrier with respect to  $a$  oscillates with the frequency that tends to infinity at  $a \rightarrow \infty$ , and at the same time its amplitude decreases as  $a^{-1}$ , in the integrals with  $\psi_E$  one can assume that  $\psi_E \approx \tilde{\psi}_E$  with a good accuracy. Such an approximation does not take into account the exponentially small probability of tunneling through the barrier  $U$  in the region of large values of  $a$ , where  $a^2 V > 1$ . It is valid for the calculations of the mean observed parameters of the Universe within its lifetime in a given quasistationary state, when this state can be considered as a stationary one. Here, we have a close analogy with the approximate description of quasistationary states in ordinary quantum mechanics (see, e.g., [35]).

In order to determine the character of motion with respect to the variable  $\phi$  we shall use the model of a scalar field which slowly (in comparison with the rapid, on average, motion with respect to the variable  $a$ ) rolls from some value  $\phi_{start}$  with the Planck energy density  $V(\phi_{start}) \sim 1$  to the equilibrium state  $\phi_{vac}$  with the energy density  $\rho_{vac} = V(\phi_{vac}) \ll 1$ <sup>1</sup>. This constant density determines the cosmological constant  $\Lambda = 3\rho_{vac}$ . At the next stage of the evolution, the scalar field oscillates with a small amplitude near  $\phi_{vac}$  under the action of quantum fluctuations. The small oscillations of the field  $\phi$  near  $\phi_{vac}$  can be quantized [37]. In such a model the motion with respect to  $\phi$  always will be finite, and the corresponding functions  $\psi_E$  will be square-integrable in the  $(a, \phi)$  minisuperspace.

## 4 Equations for Mean Values

Performing the averaging over the normalized state (3), where  $\psi_E \approx \tilde{\psi}_E$ , from Eq. (4) we obtain

$$\left\langle \frac{1}{a^4} \hat{\pi}_a^2 \right\rangle = \left\langle \frac{2}{a^6} \hat{\pi}_\phi^2 \right\rangle + \langle V \rangle + \left\langle \frac{E}{a^4} \right\rangle - \left\langle \frac{1}{a^2} \right\rangle. \quad (12)$$

In order to reduce this relation to the form which will make it possible to compare it with the Einstein-Friedmann equation for the  $\binom{0}{0}$  component of classical cosmology, we assume that in the classical approximation the wave packet represents the Universe with the scale factor  $a_{classic}$  being equal to the mean value  $\langle a \rangle$  in the state  $\Psi$ ,

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<sup>1</sup>The analogous model of a scalar field was considered for the first time in connection with the inflationary scenario (see, e.g., [2, 36] and references therein).

and the change of position of the packet in time in minisuperspace (the expansion or contraction of the Universe in accordance with the increasing or decreasing of the scale factor) obeys the laws of classical cosmology in the limiting case of zero size of the packet. In agreement with this assumption the Hubble constant will be determined by the following relation

$$H = \frac{1}{\langle a \rangle} \frac{d\langle a \rangle}{dt}. \quad (13)$$

At such a definition the problems, related with a fact that the operators  $\hat{\pi}_a$  and  $a$  do not commute between themselves, do not appear.

Let us extract the contributions from the deviations of  $a$  from the mean value  $\langle a \rangle$  in an explicit form. To this end we introduce the operators  $\xi$  and  $d\xi'/dt$ , such that

$$a = \langle a \rangle + \xi, \quad \frac{da}{dt} = \frac{d\langle a \rangle}{dt} + \frac{d\xi'}{dt}. \quad (14)$$

Then the relation (12) may be reduced to the form

$$H^2 = \bar{\rho} - \frac{\bar{k}}{\langle a \rangle^2}, \quad (15)$$

where we denote

$$\begin{aligned} \bar{\rho} &= \left\langle \left( 1 + \frac{\xi}{\langle a \rangle} \right)^{-2} \left( 1 + \frac{d\xi'}{d\langle a \rangle} \right)^2 \right\rangle^{-1} \times \\ &\times \left\{ \left\langle \left( 1 + \frac{\xi}{\langle a \rangle} \right)^{-6} \hat{\pi}_\phi^2 \right\rangle \frac{2}{\langle a \rangle^6} + \langle V \rangle + \right. \\ &+ \left. \left\langle \left( 1 + \frac{\xi}{\langle a \rangle} \right)^{-4} \right\rangle \frac{E}{\langle a \rangle^4} \right\}, \\ \bar{k} &= \left\langle \left( 1 + \frac{\xi}{\langle a \rangle} \right)^{-2} \left( 1 + \frac{d\xi'}{d\langle a \rangle} \right)^2 \right\rangle^{-1} \times \\ &\times \left\langle \left( 1 + \frac{\xi}{\langle a \rangle} \right)^{-2} \right\rangle. \end{aligned} \quad (16)$$

This equation is an exact expression. It takes into account all quantum corrections with respect to the deviation  $\xi$ . In zero approximation  $\xi = 0$ , and the change of the mean  $\langle a \rangle$  in time  $t$  is determined by the equation

$$H^2 = \langle \rho \rangle - \frac{1}{\langle a \rangle^2}, \quad (17)$$

where

$$\langle \rho \rangle = \frac{2}{\langle a \rangle^6} \langle \pi_\phi^2 \rangle + \langle V \rangle + \frac{E}{\langle a \rangle^4}. \quad (18)$$

This equation may be considered as the Einstein-Friedmann equation in terms of mean values. The quantity  $\langle \rho \rangle$  gives the mean total energy density in the Universe filled with the scalar field and the relativistic matter.

In accordance with the correspondence principle which establishes an agreement between the quantum and classical descriptions of the physical system (see, e.g., [22]), in Eq. (17) the mean values should be calculated in the state with large quantum numbers. Such a state is described by the wavefunction  $\psi_E$  with separable variables,

$$\psi_E(a, \phi) = \varphi_n(a) f_{ns}(\phi). \quad (19)$$

(An explicit form of  $\varphi_n$  and  $f_{ns}$  is given in [20, 21] for  $\phi_{vac} = 0$  and in [38] in the general case.) Here, the quantum number  $n$  describes the number of elementary quantum excitations of the vibrations of oscillator which characterizes a variation of the metric (their number is equal to  $N = 2n + 1$ ), and  $s$  characterizes the number of the elementary quantum excitations of vibrations of the scalar field near the equilibrium state  $\phi_{vac}$ . The latter excitations can form an invisible energetic component in the total energy density in the Universe [37].

The mean density (18) in the state (19) equals

$$\langle \rho \rangle = \gamma \frac{M}{\langle a \rangle^3} + \rho_{vac} + \frac{E}{\langle a \rangle^4}, \quad (20)$$

where  $\gamma = 193/12$  is a numerical coefficient which appears in the calculation of expectation values of the operators of the kinetic and potential parts of the energy density of the scalar field in expression (18),  $M = m(s + 1/2)$  can be interpreted as the amount of matter/energy in the Universe represented in the form of a sum of the elementary quantum excitations of vibrations of the field  $\phi$  with the masses  $m = ([\partial_\phi^2 V]_{\phi_{vac}})^{1/2}$ .

## 5 Quantum Corrections

Let us calculate the quantum corrections to the classical density (20), using the exact expression for  $\bar{\rho}$  from (16). We assume that  $d\xi'/dt \ll d\langle a \rangle/dt$ . This corresponds to the case, when the deviation  $\xi$  depends weakly on the mean value  $\langle a \rangle$  (i.e., the corresponding statistical distribution slowly changes in the form during the small time intervals). According to Eq. (15) the quantity  $\bar{\rho}$  can be considered as the energy density which takes into account the quantum corrections. For the states (19) it can be reduced to the form

$$\bar{\rho} = \frac{1}{Z_2} \left\{ Z_6 \frac{2}{\langle a \rangle^6} \langle \pi_\phi^2 \rangle + \langle V \rangle + Z_4 \frac{E}{\langle a \rangle^4} \right\}, \quad (21)$$

where we denote

$$Z_l = \left\langle \left( 1 + \frac{\xi}{\langle a \rangle} \right)^{-l} \right\rangle. \quad (22)$$

The quantities  $Z_l$  play the role of the “renormalization constants”. They may be rewritten in the form of the infinite alternating series

$$Z_l = 1 + \sum_{\mu=2}^{\infty} (-1)^{\mu} \frac{l(l+1) \cdots (l+\mu-1)}{\mu!} \frac{\langle \xi^{\mu} \rangle}{\langle a \rangle^{\mu}}. \quad (23)$$

Here, the mean  $\langle \xi^2 \rangle = \langle a^2 \rangle - \langle a \rangle^2$  is the dispersion, and  $\langle \xi^{\mu} \rangle = \langle (a - \langle a \rangle)^{\mu} \rangle$  at  $\mu > 2$  determines the moment of order  $\mu$  of probabilistic distribution of a scale factor with respect to its mean value  $\langle a \rangle$ . For the states with  $n \gg 1$  we find

$$\begin{aligned} \frac{\langle \xi^{\mu} \rangle}{\langle a \rangle^{\mu}} &= \frac{1}{\mu+1} \quad \text{for even numbers of } \mu, \\ \langle \xi^{\mu} \rangle &= 0 \quad \text{for odd numbers of } \mu. \end{aligned} \quad (24)$$

In this case the constants  $Z_l$  will be given by the asymptotic series

$$Z_l = 1 + \sum_{\mu=2}^{\infty} \frac{l(l+1) \cdots (l+\mu-1)}{(\mu+1)!}, \quad (25)$$

where the prime near the summation sign means that the summation is performed only with respect to the even numbers of  $\mu$ . For the Universe in the states with  $n \gg 1$  and  $s \gg 1$  from (21) we obtain

$$\bar{\rho} = \left( 1 + \frac{\Delta \rho}{\langle \rho \rangle} \right) \langle \rho \rangle, \quad (26)$$

where the quantum correction

$$\begin{aligned} \Delta \rho &= \left[ \left( \frac{Z_6}{Z_2} - 1 \right) 16 + \left( \frac{1}{Z_2} - 1 \right) \frac{1}{12} \right] \frac{M}{\langle a \rangle^3} + \\ &+ \left( \frac{1}{Z_2} - 1 \right) \rho_{vac} + \left( \frac{Z_4}{Z_2} - 1 \right) \frac{E}{\langle a \rangle^4} \end{aligned} \quad (27)$$

takes into account the contributions from the dispersion and all nonzero moments  $\langle \xi^{\mu} \rangle$  into the dynamics of the Universe.

In the case, when the contributions from the vacuum and relativistic matter may be neglected,

$$\rho_{vac} \sim 0 \quad \text{and} \quad \frac{E}{\langle a \rangle^4} \sim 0, \quad (28)$$

the relative correction to the density  $\langle \rho \rangle$  is expressed only in terms of the renormalization constants  $Z_l$ ,

$$\frac{\Delta \rho}{\langle \rho \rangle} = \frac{1}{\gamma} \left[ \left( \frac{Z_6}{Z_2} - 1 \right) 16 + \left( \frac{1}{Z_2} - 1 \right) \frac{1}{12} \right]. \quad (29)$$

Table 1: The deviation of  $\Omega$  from unity depending on the number of terms which are taken into account in the sum over  $\mu$  in Eq. (25);  $\mu_{max}$  is the largest order of the moments  $\langle \xi^\mu \rangle$  taken into account in the correction (27). The cosmological constant  $\Lambda$  was determined according to the type Ia supernovae data [39].

$\mu_{max}$	$\Omega - 1$		$-\Lambda \times 10^{58}, \text{ cm}^{-2}$
	$\Lambda = 0$	$\Lambda \neq 0$	
0	$6.63 \times 10^{-2}$	$3.47 \times 10^{-2}$	1.11
2	$1.59 \times 10^{-2}$	$8.30 \times 10^{-3}$	$2.78 \times 10^{-1}$
4	$5.68 \times 10^{-3}$	$2.95 \times 10^{-3}$	$1.00 \times 10^{-1}$
6	$2.53 \times 10^{-3}$	$1.31 \times 10^{-3}$	$4.48 \times 10^{-2}$
8	$1.29 \times 10^{-3}$	$6.72 \times 10^{-4}$	$2.29 \times 10^{-2}$
10	$7.28 \times 10^{-4}$	$3.79 \times 10^{-4}$	$1.29 \times 10^{-2}$
12	$4.42 \times 10^{-4}$	$2.30 \times 10^{-4}$	$7.84 \times 10^{-3}$
14	$2.83 \times 10^{-4}$	$1.47 \times 10^{-4}$	$5.03 \times 10^{-3}$

In accordance with Eq. (15) the density parameter  $\Omega$  at  $\bar{k} = 1$  is determined by the expression

$$\Omega = \frac{\bar{\rho}}{H^2}. \quad (30)$$

Then, taking into account (26), from (15) we obtain

$$\Omega = \left[ 1 - \frac{1}{\langle a \rangle^2 \langle \rho \rangle} \left( \frac{1}{1 + \frac{\Delta \rho}{\langle \rho \rangle}} \right) \right]^{-1}. \quad (31)$$

There exists the constraint equation  $\langle a \rangle = M$  between the geometry and matter in the approximation (28). This condition is the particular case of a more general feedback coupling relation between the geometric and energetic characteristics of the Universe

$$\langle a \rangle = M + \frac{E}{4\langle a \rangle} + 4\langle a \rangle^3 \rho_{vac}, \quad (32)$$

where the second term on the right-hand side describes the energy of a relativistic matter, while the third term gives the contribution from the vacuum of the scalar field. It follows from the condition on eigenvalues  $E$  of Eq. (4) for the states with  $n \gg 1$  and  $s \gg 1$ ,

$$E = 2N - (2N)^2 \rho_{vac} - 2\sqrt{2N} M, \quad (33)$$

where  $N = 2n + 1$ , and the mean  $\langle a \rangle = \sqrt{N/2}$  [26]. This equation must be taken into account in the calculations of the expectation values of observed parameters.

In Table 1 we give the deviation of  $\Omega$  from unity for different approximations with respect to the constants  $Z_l$ , which take into account the terms up to the moment of order  $\mu_{max}$  in the sum over  $\mu$  in (25). For example,  $\mu_{max} = 0$  corresponds to

the case  $Z_l = 1$  and is described by a zero approximation (17), (20). The value  $\mu_{max} = 2$  corresponds to the case, when one term (dispersion) with  $\mu = 2$  is taken into consideration,  $\mu_{max} = 4$  accounts for two terms (dispersion and fourth moment) with  $\mu = 2, 4$ , and so on. The column with  $\Lambda = 0$  corresponds to the condition (28), the cosmological constant  $\Lambda \neq 0$  was determined according to the type Ia supernovae data [39]. It is interesting to note that for  $\rho_{vac} = 0$  taking the dispersion into account leads to the value  $\Omega = 1.016$  that is in good agreement with the WMAP data. The astrophysical data obtained previously,  $\Omega = 1 \pm 0.12$  [6],  $\Omega = 1.02 \pm 0.06$  [7],  $\Omega = 1.04 \pm 0.06$  [8],  $\Omega = 0.99 \pm 0.12$  [9], are described by a zero approximation.

Let us consider the case, when  $\rho_{vac} \neq 0$ , but the contribution from the relativistic matter will be neglected as before. We determine a single free parameter of the theory  $\Lambda$  from a  $\chi^2$  statistic for the distance modulus of the source as a function of the cosmological redshift  $z = a_0/\langle a \rangle - 1$ , where  $a_0$  is the scale factor at the moment of observation. We take 156 type Ia “gold” supernovae as the sources with the different  $z$  [39]. The results of such analysis with  $\chi^2_{dof} = 1.17$  are given in Table 1. They demonstrate that the cosmological constant  $\Lambda$  in this theory is negative in all approximations. While one takes into account the contributions from quantum corrections of higher and higher orders of  $\mu$ , it diminishes. At the same time the scale factor  $a_0$  grows so that the value  $a_0^2 \Lambda$  remains almost constant being close to the limiting value  $a_0^2 \Lambda = -0.692$  for  $\mu_{max} \geq 10$ .

We note that the idea of occupied levels with negative energy [40] leads to a negative energy density as well [41]. Moreover, superstring models of quantum gravity which invoke compactified higher spatial dimensions are incompatible with the positive cosmological constant of the model with the cold dark matter and prefer models with negative or no cosmological constant [5].

## 6 An Asymptotic Limit of the Spatial Geometry

Since the renormalization constants  $Z_l$  are described by the asymptotic series (25) which give a finite result in every approximation, then, generally, in the limit which takes into account the moments of arbitrarily large but finite orders  $\langle \xi^\mu \rangle$ , we obtain  $\Omega = 1 + \varepsilon$ , where  $\varepsilon \sim +0$ . In other words, the quantum model predicts an arbitrarily small but finite excess of the density  $\Omega$  over unity in the homogeneous and isotropic Universe. This agrees with the basic premise (Eq. (4) describes the spatially closed Universe). As was noted in Introduction, the data of the CMB anisotropy observations most likely point out a small enough but systematic excess of the current energy density in the Universe over its critical density [10, 11].

In the limit  $\mu \rightarrow \infty$  (for an infinitely large number of terms of the asymptotic series (25)) we obtain an exact expression,  $\Omega = 1$ . This means that from the standpoint of the quantum description the Universe will be spatially flat in the epoch, when arbitrarily large, on average, deviations of the scale factor  $a$  from the mean value  $\langle a \rangle$  are possible. The assumption that the early Universe must obey the

quantum laws to a greater extent than the classical ones seems justified. Then from general physical reflections it is clear that in the early epoch, when nevertheless the state of the Universe may be characterized by the large quantum number  $n$ <sup>2</sup>, such deviations are most probable.

This result agrees completely with the conclusions of general relativity that the early Universe must be spatially flat to a higher accuracy, than nowadays<sup>3</sup>. Thus the quantum model points out the natural mechanism of fine-tuning of the parameter  $\Omega$  to unity at early stages of the evolution of the Universe, as general relativity demands, and the reason for a small possible difference of the energy density from the critical value in process of subsequent expansion.

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<sup>2</sup>Since the motion with respect to the variable  $a$  is described by an oscillator, then  $\langle a \rangle \sim \sqrt{n}$ . From this it follows, in particular, that in the state with  $n \sim 10$  the “radius” of the Universe is still close to the Planck value  $\langle a \rangle \sim 1$ .

<sup>3</sup>Estimations within general relativity give the values  $|\Omega - 1| \sim 10^{-60}$  for  $t \sim 10^{-44}$  s,  $|\Omega - 1| \sim 10^{-20}$  for  $t \sim 10^{-10}$  s, and  $|\Omega - 1| \sim 10^{-1} - 10^{-2}$  for the current epoch with  $t \sim 10^{10}$  years [2, 3].

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